NOVEMBER 13TH, 1848.

REV. HUMPHREY LLOYD, D. D., PRESIDENT, in the Chair.

THE Secretary read the following communications relative to recent antiquarian discoveries; one, a letter from Mr. Richard Young of Island-bridge, accompanying specimens of ancient Danish weapons, discovered by the workmen in excavating near the Terminus of the Great Southern and Western Railway. They consisted of a sword, much larger than has been yet found, and a smaller weapon of the same kind, together with an iron spear or pike-head, and a number of iron arrow-heads. The writer stated that he is about opening a gravel pit, which, it is supposed, may contain skeletons and antique remains. There were also presented an iron Roman sword, found in a cemetery at Treves, and an ancient urn, dug out of an old wall recently thrown down at St. Audoen's Church, together with some old coins, sent by the Rev. James Howie. A fragment of woollen fabric, worn by the ancient Irish, was presented by Sir Erasmus Burrowes of Lauragh, near Portarlington.

A vote of thanks was passed to the contributors of these interesting specimens.

Captain Larcom, V. P., having been called to the Chair, the President read a Paper "on the Corrections required in the Measurement of the Magnetic Declination."

The chief source of error in the measurement of the magnetic declination is that which arises from the torsion of the suspension thread. The angle of torsion appears to be altered, not only by the winding up of the suspension thread, which is occasionally necessary, but also by every removal of the magnet itself, the fibres of which the thread is composed appearing to

re-arrange themselves when the suspended load is withdrawn. It is also subject to changes, although to a much smaller extent, arising from hygrometric variations in the atmosphere. It is important, therefore, that we should possess a simple and accurate method of determining its amount.

Let us conceive, with Gauss,* two horizontal diameters of the suspension thread,—one at the lower extremity, parallel to the magnetic axis of the suspended magnet, and therefore moveable along with it; the other at the upper extremity, parallel to the former in the state of detorsion. The angles contained by these lines with the magnetic meridian being denoted, respectively, by u and v, the angle of torsion is $v-u$; and the moment of the force of H torsion is $(v-u)$, H being a constant coefficient. This is resisted by the earth's magnetic force, the moment of which is $mX \sin u$, or mXu, *q. p.*, the angle u being small; and therefore the equation of equilibrium is

$$H(v-u) = mXu.$$

Hence

$$v = \left(\frac{mX}{H} + 1\right) u.$$

The value of the coefficient, $\frac{mX}{H} + 1$, is determined experimentally, by observing the readings of the scale attached to the magnet, corresponding to two positions of the arm of the torsion circle connected with the upper extremity of the suspension thread. Let v_1 and v_2 denote the values of v in the two positions; u_1 and u_2 the corresponding values of u; then denoting the coefficient for abridgment by p,

$$v_1 = pu_1, \quad v_2 = pu_2.$$

Whence, subtracting and dividing,

$$p = \frac{v_1 - v_2}{u_1 - u_2};$$

* *Intensitas Vis Magneticæ Terrestris ad mensuram absolutam revocata.*

in which $v_1 - v_2$ is the angle contained between the two positions of the arm of the torsion circle, and is therefore known; and $u_1 - u_2$ is the difference of the observed scale-readings converted into angular value.

The value of $u_1 - u_2$, in this expression, must be corrected for the actual changes of declination which take place in the interval of the two readings; or else the observations must be instituted in such a manner as to eliminate, of themselves, these changes. The former course is that recommended by Gauss, and usually followed, the actual changes of declination being determined by simultaneous observations with an auxiliary apparatus. But in this, and in all similar cases in which the interval of the observations is small, the effect of such changes may be eliminated with more certainty by repeating the readings alternately in an opposite order for a few successions. Thus the errors arising from a want of exact correspondence either in the movements, or in the times of observing the two instruments, are avoided.

In order to determine the deviation of the plane of detorsion, v, the coefficient p must be altered, so as to change the value of u, while that of v is unchanged. The usual course adopted for this purpose is to diminish the magnetic moment, m, by substituting a weaker magnet. The value of the altered coefficient is to be determined experimentally in the manner already described: let it be denoted by p', and let u' be the new angle which the magnetic axis forms with the magnetic meridian. Then δ denoting the angle which the magnetic axis of the second bar forms with the lower diameter of the suspension thread, the angle of torsion is $v - u' + \delta$; and the equation of equilibrium is $v + \delta = p'u'$. Eliminating v between this and the original equation,

$$p'u' = pu + \delta.$$

Now δ is a small angle, of the same order of magnitude as u

and u', and may therefore be neglected in comparison with pu and $p'u'$. Hence, approximately,

$$p'u' = pu.$$

But, if a and a' denote the angles which the magnetic axes of the two magnets form with the line of collimation of the observing telescope, supposed fixed,

$$u' - u = a' - a\,;$$

and eliminating u' between this and the preceding equation, the error in the position of the magnet is

$$u = \frac{p'\,(a' - a)}{p - p'}.$$

Finally, the error of the plane of detorsion is

$$v = \frac{pp'\,(a' - a)}{p - p'}.$$

The angles a and a' are given by the formulæ

$$a = k\,(n - n_0), \quad a' = k'\,(n' - n_0')\,;$$

n and n' denoting the actual readings of the scales of the two magnets, and n_0 and n_0' the readings corresponding to the zero-points.

It appears that the method above described, in which the value of p is altered by the substitution of a weaker magnet, is only approximate. But a much weightier objection to it is, that the plane of detorsion, and therefore the angle v, is liable to be altered by the removal of the magnet; and thus the assumption upon which the value of that angle is inferred fails altogether.

It is easy to avoid both these sources of error. It is obvious that the value of p may be diminished by increasing H, as well as by diminishing m; and that the effect upon the angle u will be the same in both cases. Now the torsion co-

efficient, H, may be increased without removing the magnet, simply by loading it with an additional weight, care being of course taken that the total weight is within the limit which the thread is capable of sustaining. The method is simpler, and easier in practice, as well as more accurate than the received one; and if the position of the magnet, when loaded and unloaded, be observed for several alternations, and in rapid succession, the result may be obtained with very great precision. The difference of the angles, $a' - a$, is, in this case, simply the difference of the observed scale-readings reduced to angular value; or,

$$a' - a = k\,(n' - n).$$

The following are the details of a series of observations made according to this plan. The last columns in the following Tables contain the differences between each reading, as given in the second columns, and the mean of the preceding and subsequent readings. The additional weight was 10 oz., being about one-half the weight of the magnet itself and its appendages.

Obs. I.			Obs. II.		
Magnet.	Reading.	Diffs.	Magnet.	Reading.	Diffs.
Unloaded	81·9		Unloaded	90·9	
Loaded	80·6	– 1·65	Loaded	88.3	– 1·9
Unloaded	82·6	– 2·0	Unloaded	89·5	– 2·4
Loaded	80·6	– 2·65	Loaded	85·9	– 2·4
Unloaded	83·9	– 2·4	Unloaded	87·1	– 2·0
Loaded	82·4	– 2·35	Loaded	84·3	
Unloaded	85·6				
Mean diff. = – 2·21			Mean diff. = – 2·18		

Hence $n' - n = -2{\cdot}20$; and $a' - a = k\,(n' - n) = -1'{\cdot}58$.

In determining the values of p and p', the arm of the torsion

circle was turned forwards and backwards, alternately, through two circumferences, and the scale-readings noted after each change. The following are the results:

I. Magnet unloaded.

$$v_1 - v_2 = 720^\circ;\ u_1 - u_2 = 29'{\cdot}31.$$

II. Magnet loaded.

$$v_1 - v_2 = 720^\circ;\ u_1 - u_2 = 43'{\cdot}43.$$

III. Magnet unloaded.

$$v_1 - v_2 = 720^\circ;\ u_1 - u_2 = 29'{\cdot}22.$$

Hence we have

$$p = 1477;\ p' = 995;\ \text{and}\ \frac{p}{p'} = 1{\cdot}484.$$

Accordingly $u = -3'{\cdot}28$; and $v = -80^\circ\ 45'$. The values of u deduced from the *separate* observations differ only by $0'{\cdot}04$.

In order to determine the effect of hygrometric changes on the torsion of the suspension thread, a weaker magnet was attached; and the air within the box was alternately moistened to saturation by wet sponges, and dried by chloride of calcium, great care being taken to render the box air-tight. The position of the magnet was observed at an interval of some hours after each change; and the actual changes of declination were, under these circumstances, eliminated by the help of simultaneous observations with an auxiliary apparatus. Finally, the observed changes were reduced in the ratio of the magnetic moments of the two magnets. The partial results presented considerable discordance, notwithstanding every care in the observations; but the final mean is probably not remote from the truth. It gave,—as the total effect upon the position of the magnet, produced by the transition from complete dryness to complete saturation,—a change of $+\ 1'{\cdot}0$, which corresponds to a change of position of the plane of detorsion of $24\frac{1}{2}$ degrees. The greatest change (from its mean state) in

the humidity of the atmosphere is probably about one-half of the total range; so that the limit of error of declination due to this cause may be considered to be $0'\cdot5$.

It will, of course, be understood that the effect here stated is that produced on the individual thread; and it is given merely as an example of the amount of error to be expected under ordinary circumstances. If the separate fibres which compose the thread were perfectly parallel, and equally strained, we have no reason to suppose that the changes of moisture would produce any change of torsion.

The next subject which claims our attention is the disturbance produced in the position of the magnet by the action of the other magnets of the Observatory, or by any other extraneous magnetic forces. The course which has been pursued at the Dublin Magnetical Observatory in determining the effect exerted by other magnets is very simple, and admits of the utmost precision. It consists in *reversing* the acting magnet (or turning its magnetic axis through 180°), and observing the new position of the magnet acted on; the difference of the two positions is *double* the error sought. In fact, the moment of the force exerted by the former magnet upon the latter is

$$\frac{mm'}{2D^3}\,\{\sin(\phi+\phi') - 3\sin(\phi-\phi')\};$$

in which m and m' denote the magnetic moments of the two magnets, D the distance of their centres, and ϕ and ϕ' the angles which their magnetic axes form with the joining line.* The value of this quantity is unaltered, although its sign is changed, when $180° + \phi$ is substituted for ϕ; and, consequently, the disturbing effect is *equal* and *opposite* to that produced in the original position of the acting magnet.

It is scarcely necessary to advert to the advantages of this course over that which is sometimes adopted, and which

* Trans. Royal Irish Academy, vol. xix. p. 163.

consists in the *removal* of the acting magnet. The change produced is doubled, and therefore the effect of errors of observation halved; and the two parts of the observation may be made at a very short interval,—an essential condition of accuracy in the elimination of the irregular changes. If the observations be repeated six or seven times in rapid succession, and at a time in which the irregular changes are small, the final result may be depended on to a few hundredths of a minute.

The following observation may serve as a favourable example of the accuracy attainable by this process. It was made to determine the effect of the action of the magnet of the balance magnetometer upon that of the declinometer, the two magnets being in the plane of the magnetic meridian, and the distance of their centres nineteen feet. The third column contains the differences of the corresponding readings in the second column, and the means of the preceding and subsequent readings.

North end.	Reading.	Differences. (N – S)
North	51·37	
South	51·78	– 0·46
North	51·28	– 0·49
South	51·76	– 0·47
North	51·30	– 0·47
South	51·78	– 0·49
North	51·28	
		Mean = – 0·48

Hence the error = – 0·24 scale divisions = – 0′·17.

This process cannot be employed when the disturbing action is that of a mass of soft iron, and we must, in that case, have recourse to the less accurate and less easy method of removal. The following will serve as an example of the mode of dealing with such cases.

A considerable portion of the iron railing, now erected on the wall in Nassau-street, had been, previously to its erection, and during the absence of the writer in the summer, laid down in a horizontal position in the College Garden, at the southern side of the Observatory. The several pieces of which the railing is composed (each fourteen feet in length, and containing twenty-eight bars) were found placed in a continuous line parallel to the south walk of the garden, and forming an angle of 59° with the magnetic meridian. This line extended from a point nearly opposite the Observatory to a distance of 255 feet, and was distant from the declinometer magnet, at the nearest point, by 153 feet. It would, of course, have been impracticable to remove this great mass of iron, and to replace it rapidly, or for many alternations. Instead of this, the effect of a *single* piece of the railing was observed at a nearer distance, from which, and from the known laws of the mutual action of magnets, the total effect was deduced by integration.

Let a denote the perpendicular from the centre of the moveable magnet upon the line of the bars; and let α be the angle which that perpendicular makes with the magnetic meridian. Then, in the expression already given for the moment of the force exerted by a fixed upon a moveable magnet, $\phi + \phi' = \alpha$, and the moment of the force exerted by a single bar is

$$\frac{mm'}{2D^3}\left\{\sin\alpha + 3\sin(\alpha - 2\phi)\right\},$$

m being the magnetic moment of the bar, and m' that of the suspended magnet. The moment of the force exerted by an element of the railing whose length is dx, is obtained by multiplying this by ndx, n being the number of bars in the unit of length. This is equilibrated by the earth's magnetic force, whose moment is

$$m'Xdu,$$

du being the change of position produced by a single element *of the railing; so that we have*

$$Xdu = \frac{mn}{2D^3} \{\sin a + 3 \sin (a - 2\phi)\, dx\}.$$

But

$$x = a \tan \phi, \quad dx = \frac{ad\phi}{\cos^2\phi}, \quad D = \frac{a}{\cos\phi};$$

and substituting, and integrating,

$$Xu = \frac{mn}{2a^2} \int \{\sin a + 3 \sin (a - 2\phi)\} \cos \phi d\phi$$

$$= \frac{mn}{4a^2} \{4 \cos (a - \phi) + \cos (a - 3\phi) - \cos (a + \phi)\},$$

and between the limits $\phi = 0$ and $\phi = \beta$,

$$Xu = \frac{mn}{2a^2} \left\{ 4 \sin \left(a - \frac{\beta}{2}\right) \sin \frac{\beta}{2} + \sin (a - \beta) \sin 2\beta \right\}.$$

The value of *mn* in this expression was obtained, as before stated, by observing the effect of a single piece of railing in a known position. For convenience of calculation, this piece was placed upon the perpendicular let fall from the centre of the magnet upon the line of the rails, the bars being parallel to their original position. In this position $\phi = 0$; and therefore the moment of the force exerted by a single bar, at the distance D, is

$$\frac{2mm'}{D^3} \sin a;$$

so that, if ε denote the disturbance produced by a portion of the railing, whose length is unity, in this position,

$$X\varepsilon = \frac{2mn}{D^3} \sin a.$$

Finally, dividing the former result by this, *mn* and X are eliminated, and we have

$$u = \frac{D^3 \varepsilon}{4a^2 \sin a} \left\{ 4 \sin \left(a - \frac{\beta}{2}\right) \sin \frac{\beta}{2} + \sin (a - \beta) \sin 2\beta \right\}$$

The piece of railing which was the subject of experiment was raised, by the help of a windlass and pulleys, into the horizontal plane containing the magnet, and was fixed in the required position at the distance of 60 feet. The position of the magnet being observed, the mass of iron was lowered, and removed to a sufficient distance upon a truck, and a fresh observation taken. This process was repeated several times in rapid succession, and a series of observations thus taken, with the magnet alternately disturbed and undisturbed. The following are the results:

Time.	Railing.	Reading.	Difference.
$12^h 45^m$	present,	60·30	
12 53	removed,	58·12	+ 2·08
1 0	present,	60·10	+ 1·80
1 7	removed,	58·48	+ 2·02
1 13	present,	60·90	+ 2·08
1 20	removed,	59·15	+ 1·86
1 25	present,	61·12	+ 1·90
1 32	removed,	59·30	

These observations are very satisfactory. They give, for the mean difference of the readings due to the presence of the railing, + 1·96 divisions of the scale = + 1′·41. Hence the length of the piece of railing being 14 feet, the effect produced by a piece whose length = 1 foot, is

$$\varepsilon = + 0'\cdot 101.$$

But $D = 60$, and $a = 153$, the unit of length being 1 foot. Also $\alpha = 31°$; and the length of the line of railing being 255 feet, $\beta = \tan^{-1}(1\cdot 67) = 59°$. Hence the quantity within the brackets, in the value of u, is equal to $-$ 0·363; whence finally,

$$u = - 0'\cdot 164.$$

The only other disturbing causes, in addition to those already noticed, are those which affect the position of the read-

ing telescope. Upon these it is unnecessary to dwell. The changes of position are to be determined by referring the telescope, from time to time, either to a distant fixed mark, or to a fixed collimator. In the Magnetical Observatory of Dublin, the telescope of the transit instrument is used as a collimator, and thus the position of the reading telescope is referred immediately to the astronomical meridian.

Sir Robert Kane laid before the Academy some specimens of the series of maps now being prepared in the Museum of Irish Industry, illustrative of the distribution of the values of land in Ireland. The principle of the construction of these maps was described by Sir Robert Kane to consist in the reduction of the numerical results of the Government valuation of Ireland, now in process of publication, under the direction of Mr. Griffith, to such system of classification, indicated by characteristic colours, as would show the manner in which the soils of different financial values are distributed over the country. The specimens laid before the Academy comprised two sets of maps, of which the one showed the registered valuation of townlands; the second, the values of groups of townlands reduced to an average of value. The method employed was the following. Sir Robert Kane, having found by consultation with experienced agriculturists that the unit of difference of value might be taken as sufficiently small for practical purposes at two shillings per statute acre, reduced the values of townlands to a scale of ascending rates, from zero to thirty-six shillings per acre, and then, having transferred to the county index maps of the Ordnance Survey the boundaries of townlands, which are engraved only on the maps of the six-inch scale, those are coloured with tints respectively indicative of the values, and thus a pictorial representation of the distribution of the different classes of land is obtained. As the map so formed becomes, however, very detailed, the number of tints very numerous and very much intermingled, and hence,